CHAPITRE 8 – RISK AND UNCERTAINITY

I – INTRODUCTION

Cost and benefit analysis requires knowing the future.

We are going to deal with:

- *expected values*, a measure reflecting risk.

- *option value*, a measure reflecting the utility of income in a risky environment.

II – EXPECTED VALUE ANALYSIS

Modelling uncertainty as risk begins with the specification of *set of contingencies*.

Contingencies

Contingencies can be thought as possible events outcomes or state of the world such that are and only one of the relevant set of possibility will actually occur.

- A set of contingencies must capture the full range of lively variations in the net benefits of the policy.
- Contingencies may be non discrete and cover all the positive outcomes between two extremums or discrete and then there are only two or more scenarios.

Figure 1 illustrates the representations of a continuous set with discrete contingencies.

The horizontal axis gives the number of inches of summer rainfall in an agricultural region.

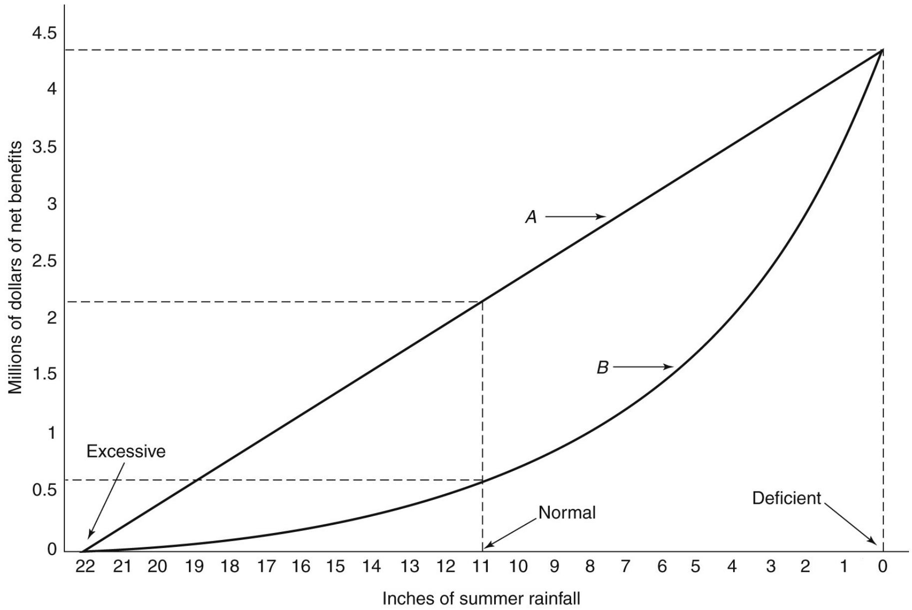
The vertical axis gives the net benefit of a water storage system, which increases with the amount of rainfall decreases.

Imagine that an analyst represents uncertainty about rainfall with only two contingencies *excessive* and *deficient*.

Excessive rainfall leads to 22 inches of rainfall and yield zero benefit for the storage system

*Deficien*t rainfall leads to zero inches of rainfall and yields 4M€ of net benefit for the storage system.

Figure 1



If the relation between rainfall and net benefit follows the straight line labelled A, and all the rainfall amounts between zero and 22 are equally likely then the average of net benefit is 2.2M$

If the two scenarios are equally likely the average net benefit is 2.2M$ so using two scenarios is enough and gives and accurate representation.

Now imagine that the net benefit follows the curved line labelled B. Again assuming that every rainfall between zero and 22 inches are equally likely, the average of net benefits over the **full continuous** range would be only by about 11M$.

Using two contingencies would grossly overstate the average net benefits from storage system.

Adding normal as contingency that ssumes 11 inches of rainfall and averaging net benefits over all three contingencies yields net beenfits of $1,6millio, which is more representative than the average calculated with two contingencies but sill considerablylarger tha $1,1 calculated over the full continuous range.

Probabilities

The next step is to assign *probability of occurrence* to each contingency.

If there are three contingencies we must assign three probabilities such that P1+P2+P3=1.

The *expected value* is

$$E[NB] = P_1(B_1 - C_1) + ... + P_n(B_n - C_n)$$

Table 1 shows the contingencies payoff for a policy building a temporary dam that provides water from irrigation.

With or without the dam, the farmer can be viewed as facing two contingencies:

- it rains a lot or not, (if it is wet the farmers will always produces more crops).

- Without the dam, the farmer receive an income of 100$, and only 50$ if it does not rain very much.

As a result of the dam, his income will increase by 50$ if it is dry but only 10$ if it is wet;

These 50$ and 10$ figures are the surplus that the farmers receive from the dam under each contingencies.

In expected values terms, assuming that dry and wet are equally likely means that the surplus is equal to 30$.

III – OPTION PRICE AND OPTION VALUE

Economist considers as *option price* the amount that individuals are willing to pay for policies prior the realisation of contingencies.

In general individuals are risk averse, expected surplus can either underestimate or overestimate option price, depending on the sources of risk.

-For risk *adverse person*, *Expected Surplus* will underestimates option price for policies that reduces risk.

Expected utility

To calculate the farmer benefit from the dam we first calculate his ex post utility (EU) without the dam.

We the find his *option price* – that is the maximum amount he would be willing to pay for the dam or equivalently, the amount that gives him the same expected utility has he would have without the dam.

To compute these amounts we need to know the farmer utility function. Normally we would not have this information, which is why, in practice, expected surplus, rather than option price is usually used.

For the sake of simplicity, let's assume that the utility function of the farmer is the natural log of his income.

	Dam	No dam
Wet	110	100
Dry	100	50
Expected Value	105	75
Variance	25	625

With: $E[X]=\sum x_i p(x_i)$ and $Var[X]=\sum (x_i - E[X])^2 p(x_i)$ with $p(x_i)$ is the probability of ith outcome occurring. In our case,

$E[X]=0.5(110)+(0.5)(100)=105$ and $Var[X]$

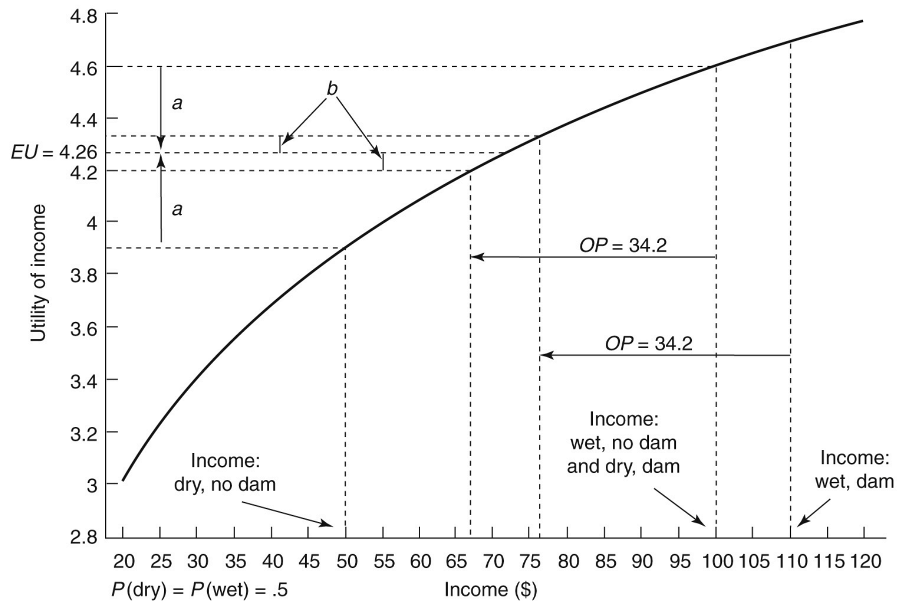
$$=(0.5)(110-105)^2+(0.5)(100-105)^2=25$$

The curved line in figure 2 shows the utility function. In the absence of dam, the farmer realises an income of 50$ if it is dry and 100$ if it is wet.

Because the farmer realizes income of $50 if it dry and $100 if it wet. Because the probabilities of wet and dry are each one half, the expected utility without the dam can be found as the point midway between the utilities of these no-dam incomes.

The point on the vertical axis labelled EU=4,26 is exactly *a* units of utility away from each of the contingent utilities. As is is midway between them, it equals the expected utility. If the dam is built, then the farmer receives an income of $100 if it is dry and $110 if it is wet.

Figure 2



The *option price* for the dam is the maximum of income that the farmer would be willing to give up to have the dam, in other words, the amount that would allow him to have the same expected utility with the dam as he would have without.

Option price

$0.5U(110-OP)+0.5U(100-OP)=EU$

$\Rightarrow EU=4.26$ and $OP=34.2$

for $U(i)=\ln(c)$ where c is the net income

The arrow marked 34.2 shift the contingent income with the dam by subtracting 34.2$ from each so that the net contingent income are 65.8$ and 75.8$. The utilities of these net incomes are each *b* away from 4.26 so that their expected utility equals the expected utilities of no dam.

Then with or without dam 34.2$ of certain payment give the farmer the same expected utility.

The farmer option price for the dam of 34.2$ exceed is expected surplus of 30$. Thus if the opportunity cost of the project were 32$ and farmers would be the only beneficially, the common practice of using expected value of surplus would result in rejecting building the dam, when in facts, the option price indicates that building would increase the farmer's utility.

Certainty line

Figure 3 presents an alterative graphical explanation of the relationship between expected surplus and option price.

The vertical axis indicates the farmer willingness to pay amount if it is dry; the horizontal line represents his willingness to pay amounts if it is wet. Thus point A represents his surplus under each contingency, 50$ if it is dry 10$ if it is wet.

Here there is a slight difference way to view point A. imagine that before the government will build the dam, the farmer would be required to sign a contract A that stipulates that he will pay the government an amount equal to X_w if it turns out to be wet and X_d if it turns out to be dry. This is called a contingent contract.

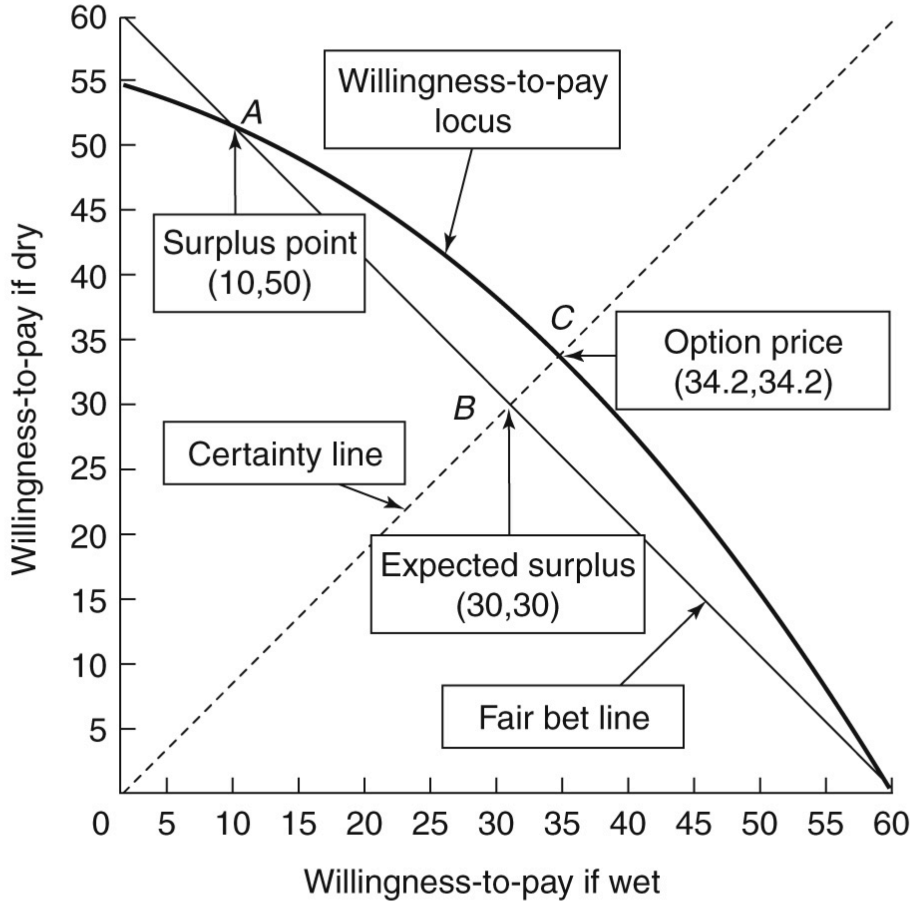


Figure 3

The concept of *contingent contract* is useful for thinking about how much the farmer values the dam.

The benefits of the dam corresponds to the maximum value that the government could assign to X_w and $_d$ and still get the farmer to sign the contract A, 10$ and 50$ respectively.

The farmer would ne willing to sign at these amounts because he would be exactly back to the situation he faced without the dam when his income equalled 100$ if it rained and 50$ it it was dry.

In others words 10$ and 50$ are his maximum willingness to pay under contract A.

Surplus point:

$U(110 - S_w) = U(100) \ then \ S_W = 10$

$U(110 - S_d) = U(50) \ then \ S_d = 50$

Expected surplus

$E(S) = 0.5S_w + 0.5S_d = 30$

Notice that the (10,50) payment measures the surplus the farmer receives because the dam increases his income but not the utility he receives because the dam reduces the risk he faces. If the farmer makes he payment his income variability would be exactly what it was without the dam.

To examine the change in risk resulting from the dam let's assume that the government suggests a contract B that allow him to pay 30$ regardless of the whether (D,W).

If the government does this the expected value of the payment he will receive would be equal under the two contracts.

However contract B would place the farmer on a line that bisects the origin of the graph. This line is called "certainty line" because payments along it are the same regardless of what contingencies occurs. Thus any point along this line incusing B represents a certainty equivalent.

Certainty line intersects another line. The one passes through the surplus point, but every point doing it has the same expected value.

For example, the expected value would always be equal to 30$ along this line. This line is called the *fair bet line*.

To see why imagine a flipping coin. A payoff $10 if you get heads and $40 if you get tails. Thus the slope of this fair bet line, -1, is equal to the negative of the ratio of the probabilities of the contingencies.

As one moves along the fair bet line toward the certainty point, the expected value remains always the same but the variation in income decreases. Finally, at point B on the certainty line, the pay off is equal regardless of which contingencies, head or tails, actually occurs.

In our example the payoff is $30.

The intersection of the fair bet line and the *certainty line* corresponds to the solution of the following equation:
$E(S) = p_1 x_1 + p_2 x_2$ (expected value line)
$x_1 = x_2$(certainity line)
$p_1 = 1 - p_2$

where x1 defines the location of points on the vertical axis and x2 on the horizontal axis. Solving these equations gives E(S)=x, where x is the point of intersection.

The willingness to pay locus

Let's ask to the farmer if he would be indifferent between signing contract A under which he must pay $10 if wet and $50 if dry and contract B under which he must pay $30 regardless of whether it is wet or dry.

Contingency	Probability	Income under contract A	Income under contract B
Wet	0.5	$100	$80
Dry	0.5	$50	$70
EV		$75	$75

Although the expected value (EV) of income under the contracts would be identical the variation in risk between two contingencies is obviously much less under contract B

Thus comparing contract A and contract B, means examining the effect of the dam on the risk facing the farmer, while holding the expected value of his income constant.

Willingness to pay locus
$\alpha_w, \alpha_d / 0.5(110 - \alpha_w) + 0.5(110 - \alpha_d) = EU$

If the farmer is risk averse and, hence, would prefer a more stable to a less stable income from year to year, the he will not e indifferent between the two contracts but will refer B to A, because with B he will v-faces less risk.

Now recall that at point A the farmer was willing to sign a contract that would require him to pay $10 of wet and $50 if dry and the expected value this expected payment was $30. Because the farmer prefers B to A this suggests that in order to reach the certainty line, the farmer would be willing to sign a contract requiring him to pay a certainty equivalent greater than $30.

The maximum such amount that he would pay is represented by point C, a point that is further north east long the certainty line than point B.

Point C represents the farmer option price, the maximum amount he would be willing to pay for both: the increase in expected income and the reduction in risk resulting from the dam. In other words, it incorporates the full value of the dam to the farmer. Conceptually, it is the correct measure of benefits that the farmer would receive from the dam.

But in CBA, point B, the expected value of the surplus, resulting from the dam-that is typically measured. Although point B captures the effect of the dam on expected income it does not incorporate the effect of the dam on income variability of risk.

Although the farmer would prefer point B to point A, he would be indifferent between B and C. indeed; a curve drawn between these points is very similar to an indifference curve. The curve, the willingness to pay locus shows all the combination od combined payments for the dam that give the farmer the same expected utility with the dam or without it.

If the cost of the project does not depend on the contingency that occurs, then it would be also represented as a point on the certainty line. If the option lies further to the northeast along the certainty line the project would increases farmer's welfare.

IV – IS OPTION PRICE THE BEST MEASURE OF BENEFITS?

Option price generally does not equal expected surplus in circumstances of risk. Is OP the correct measure?

If complete and actuarially fair insurance is unavailable against the relevant risks, then the larger of OP and ES is the conceptually correct measure of benefits.

Insurance is complete if a person can buy enough insurance to eliminate all risk. It is actuarially fair if the price depends only on the true probabilities of the relevant contingencies. Availability of actuarially fair insurance means individuals could move from the initial point in contingent claims space along the fair bet line through the purchase of insurance. The availability of complete insurance allows individuals to move all the way to the certainty line.

Problems with Insurance

Moral hazard (changes in behaviour induced by insurance coverage) and adverse selection (insurees have better information on risks than insurers) limit the availability of actuarially fair and complete insurance. Other limitations arise because: insurers are unwilling to insure unique assets that are not easily valued in markets; pooling risk groups makes some pay an actuarially unfair price; limiting coverage of certain risk groups means complete insurance is unavailable; some risks are so correlated (i.e., all happen together) that pooling risk does not sufficiently reduce risk to allow actuarially fair prices.

Determining the bias in expected surplus: signing option value

Option Value was initially interpreted as a separate benefit category. It is more accurate to interpret it as the bias in benefits resulting from measuring by expected surplus rather than option price. Specifically,

$$OV = OP - E(S).$$

Determining the Sign of OV

General heuristic: for risk averse individuals and normal (inferior) goods, treat OV as negative (positive) for income uncertainty, ambiguous for other demand-side uncertainties, and generally positive (negative) for supply side uncertainties. It is not possible to quantify OV using information from which estimates of expected surplus are typically made.

Rationales for expected surplus as a practical benefit measure

Expected Values and Aggregate Social Benefits

If society were risk neutral, then choosing policies that individually maximize expected NB would be efficient in the sense of maximizing the expected value of society's portfolio of policies. If costs and benefits are spread broadly over a large population, then the effect on an individual's income is likely to be small. Risk averse people can be approximated as risk neutral in such situations. Therefore, aggregation of individual preferences would lead to risk neutrality at the social level so that ES would be an appropriate measure of benefits. Variable magnitudes (i.e., large) and uneven distribution of costs and benefits (targeting specific groups), however, weaken the argument.

Related argument: assume that society holds a fully diversified portfolio of policies that allows it to self-insure against the risks of particular projects (i.e., pool risk across projects so it effectively has complete and actuarially fair insurance). Then the larger of either the OP or ES is the appropriate measure. Therefore, benefits would always be at least as large as ES, so any project with positive NB would be potentially Pareto improving. This argument relies on the aggregation of NB across policies so that the potential Pareto criterion can be met overall (as opposed to averaging costs and benefits across individuals). The weakness of diversification is that it does not eliminate all risk, and does not permit fully effective self-insurance. Therefore, it does not provide a fully satisfactory rationale.

Expected Values and Pooling Risks across Individuals: Collective and Individual Risks

Collective Risk: the same contingency will result for all individuals in society. (Realized NB can substantially differ from expected NB.)

Individual Risk: the contingency realized by each individual is independent of the contingency realized by any other individual.

The process of averaging risk tends to produce results of NB close to those calculated by the ES procedure. It also means the larger of OP and ES is the appropriate benefit measure, which would be potentially Pareto improving. It will not, however, necessarily lead to the most efficient policy in comparison to mutually exclusive alternatives.